

# Updating the Discount Rate Used for Benefit-Cost Analysis at Seattle Public Utilities – DRAFT

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## Summary

Most people – or at least most economists – agree that some form of discounting is required to properly compare dollars in the future to dollars in the present. However, there's much less agreement on what the correct discount rate is or how it should be determined. Over the past decade, the official policy of Seattle Public Utilities has been to use a real<sup>1</sup> discount rate of 5% for benefit-cost analysis (with low and high values of 3% and 7% for sensitivity analysis). In recent years, SPU economists have begun to question whether this is the correct discount rate, and specifically, if it is too high.

The value of the discount rate can have a considerable impact on the outcome of benefit-cost analyses. A high discount rate places a low value on costs and benefits in the future relative to the present. Thus, a project with high initial costs but benefits only accruing in the distant future will look much less attractive when evaluated at a high discount rate than at a low one. Just the opposite is the case when there are significant short term benefits but costs are incurred later. The choice of discount rate can play a big part in determining which projects look like a good idea and which ones do not.

A review of the literature has identified a number of different methods for determining the discount rate. While much of the literature and many text books only discuss the discount rate in conceptual terms, a few authors do provide some practical advice on what number to use. These range from 1% to 10% with most of them concentrating in the 1.5% to 4% range. This paper summarizes the various approaches to determining the appropriate discount rate and recommends a policy for SPU. The recommendation is that SPU adopt a real discount rate of **2.2%** for benefit-cost analysis with low and high values of **1.5%** and **3.5%** for sensitivity analysis. It is also recommended that project risk be dealt with separately from the discount rate through defining uncertainty bounds around the estimates of a project's costs and benefits.

## Background

Benefit-cost analysis requires a common unit of measure. The problem is the standard economist cliché, “you can't compare apples to oranges.”<sup>2</sup> Before all the costs can be totaled and subtracted from the sum of all the benefits, everything must be expressed in the same units and the traditional choice of units is dollars. Ideally, all benefits and costs can be quantified in dollar terms but in reality, that is often not the case. Even if that can be accomplished however, there always remains at least one more step because not all dollars are equal. A dollar a year from now is not worth as much as a dollar now. A dollar 10 years from now is worth even less. Why? There are at least three reasons: time preference, the opportunity cost of capital, and

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<sup>1</sup> i.e., adjusted for inflation.

<sup>2</sup> Actually, you *can* compare apples and oranges but you need common units before you add them.

risk.<sup>3</sup> If you ask me for \$100 now and promise to pay me the \$100 back in a year, I'm going to say no. If I give you that \$100, I'll lose the option of using it myself to buy something I need or want right now. I'll also lose the option of saving that \$100 and earning interest. Finally, there's the risk that you will not pay me back as promised. To get that \$100 from me now, you'll have to promise to pay me back more than \$100 a year from now. The extra amount that must be paid, expressed as a percentage, is the discount rate. If I'm just willing to give you \$100 in exchange for your promise to repay me \$105 in one year, my discount rate is 5%.<sup>4</sup> In other words, \$100 a year in the future is worth 5% less to me than it is now.

This concept is used to convert the dollar value of benefits and costs that will occur in various future years to "present values," a common unit of measure that allows "apples to apples" comparison. The present value of a future benefit is equal to its future value in real dollars reduced by the discount rate as many times as the number of years in the future the benefit will occur.<sup>5</sup> Calculating present values is one of the necessary tasks in conducting a benefit-cost analysis, but it requires the choice of an appropriate discount rate. The choice is extremely important because different discount rates can produce very different results in a benefit-cost analysis. For example, a project with significant initial costs followed by a stream of benefits over time may yield a positive Net Present Value (NPV) with a low discount rate but a negative NPV with a high discount rate. The goal of discounting is to compare benefits and costs that occur at different times based on the rate at which society is willing to make such tradeoffs.

Unfortunately, little agreement exists among economists as to what the correct discount rate is. Controversy surrounds the proper conceptual definition of the discount rate as well as its numerical value. In what follows, many different approaches to determining the discount rate are described along with their implications regarding a numerical value. The discussion is organized as follows:

- Market-Based Interest Rate Approaches
  - Social Marginal Rate of Time Preference
  - Social Opportunity Cost of Capital
    - Shadow Price of Capital
    - Weighted Average
  - Government Borrowing Costs
- Non-Market Based Approaches and Other Considerations
  - Optimal Growth Rate Approach
  - Intergenerational Discounting: Constant vs. Time-Declining Discount Rates
  - Gamma Discounting – A Survey Approach

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<sup>3</sup> There's actually a fourth reason - inflation - that reduces the worth of future dollars, but this is handled differently. Economists adjust for inflation by using "constant" or "real" dollars as their unit of measure. Constant dollars are dollar amounts that have been adjusted by means of price and cost indices to eliminate the effects of price inflation and allow the direct comparison of dollar values across years.

<sup>4</sup> This is a "real" discount rate with no inflation premium because it's associated with "real" dollars.

<sup>5</sup> In mathematical terms, the present value (PV) is equal to the future value (FV) divided by  $(1+r)$  to the  $t$  power, where  $r$  equals the discount rate and  $t$  equals how many years in the future the benefit or cost occurs:

$$PV = \frac{FV}{(1+r)^t}$$

- Risk and Uncertainty
- Discount Rates Used by Other Government Agencies
- Convergence/Conclusions/Recommendations

## Market-Based Interest Rate Approaches

How does society choose how much to consume now versus how much to invest now so it can consume later? Many economists assume that it is many individuals interacting in the market with their decisions to lend or borrow that should determine the *social* discount rate.<sup>6</sup> Of course different individuals have different preferences about the timing of present and future consumption. But in a “perfect” world – at least as defined by economists – there are no taxes, no transactions costs, and no externalities; there are perfect capital markets, everybody has access to the same information, and people’s behavior is “rational” and consistent. In this world, the different rates implied by competing concepts of the appropriate discount rate converge on a single equilibrium value. Individuals can lend (postpone consumption) or borrow (accelerate consumption) at the same interest rate, which also equals the rate at which government can borrow and the *marginal* rate of return on private investment<sup>7</sup>. Equilibrating supply and demand for loanable funds ensures that these rates are all equal at the margin. This market-clearing interest rate represents both the marginal social rate of time preference, the marginal rate of return on private investment<sup>8</sup> AND the cost of government borrowing. As such, it arguably represents the most appropriate choice of discount rate for calculating present values.

The real world is far less convenient, however. Taxes, transaction costs, and unequal access to capital markets and information, to name a few, all drive a wedge between the social rate of time preference (i.e., the rate at which society is willing to postpone a marginal unit of current consumption in exchange for future consumption) and the marginal rate of return on private investment. Just taking the issue of taxes, the social opportunity cost of capital is the marginal rate of return on private investment measured *before taxes*. However, the social rate of time preference can be approximated by the interest earned by individuals on their savings *after taxes*. Thus, these two different concepts for determining the appropriate discount rate produce different numerical values.

*Social Marginal Rate of Time Preference (SMRTP)*. There’s little evidence that people behave in a way that economists consider rational and consistent, making it difficult to pin down an unambiguous social rate of time preference. Some people save money at very low rates of interest while others borrow at high rates and still others do both at the same time. A single individual may be simultaneously paying down a mortgage, investing in the stock market and borrowing on credit cards. Research has demonstrated that people appear to use different rates to discount large versus small amounts, losses versus gains, choices involving the near future as against choices farther out in time, and choices between speeding up versus delaying consumption. Depending on the type of inter-temporal choice being made and who’s making it, a very wide range of marginal rates of time preference can be inferred from observing people’s

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<sup>6</sup> Note that the focus is now specifically on what rate of discount is most appropriate for society to use in making public investment decisions. This is the *social* discount rate.

<sup>7</sup> In the absence of differences in risk. Risk is discussed in more detail below.

<sup>8</sup> i.e., the social opportunity cost of capital

behavior in the capital market. For these and other reasons, the social rate of time preference is not directly observable and may not equal any particular market rate.

All of the above is enough to make economists throw all three of their hands<sup>9</sup> up in frustration. A common approach<sup>10</sup> to estimating the social marginal rate of time preference however has been to ignore all these confounding considerations and assume that the best return that most people can earn in exchange for postponing consumption is the real after-tax return on savings – approximated by using the market rate of interest from long-term, risk-free assets such as government bonds net of taxes. Using inflation-adjusted interest rates on 10-year treasury bonds since 1954 and calculating the after-tax return produces a discount rate of about **1.5%**.

**Table 1 – Annual Rates of Return on 10-Year Treasury Bonds**

	10-Year Treasury Constant Maturity Rate		
	Annual Rates of Return: 1954-2010		
	Nominal	Real	After Tax
Average	6.3%	2.6%	<b>1.5%</b>
Maximum	13.9%	8.1%	4.8%
Minimum	2.4%	-3.5%	-1.7%

Source: Board of Governors of the Federal Reserve System

*Social Opportunity Cost of Capital (SOCC)*. The social opportunity cost of capital is equated with the Marginal Rate of Return on Private Investment. One argument for the rate-of-return approach is that the most valuable alternative use of public investment funds is private investment, not private consumption. Because capital income is taxed, private investors focus on projects expected to return enough not only to justify their delayed gratification but also to pay the required taxes. The Federal Office of Management and Budget takes this approach, identifying its standard rate of 7 percent (in real dollars) as "the marginal pretax rate of return on an average investment in the private sector."<sup>11</sup> There are some problems with this 7% figure though. One is that it's old – the circular has not been updated since 1992.<sup>12</sup> From several sources of historical data on total returns to stock<sup>13</sup>, the average annual real rate of return over long periods of time appears to be a little lower, about 6.5%.

A much greater concern is that this may not be the correct concept. Boardman et al, in their textbook on cost benefit analysis<sup>14</sup> strongly suggest that the best proxy for the marginal rate of return on private investment is the real, before-tax rate of return on corporate bonds. They cite four reasons for using the bond rate rather than the average return on equities, the most

<sup>9</sup> The one hand, the other hand, and the invisible hand.

<sup>10</sup> Boardman, Anthony E., Greenberg, David H., Vining, Aidan R., and Weimer, David L., 2006, Cost Benefit Analysis: Concepts and Practice, Pearson Education Inc. – Prentice Hall, p 250-251.

<sup>11</sup> Office of Management and Budget, "Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs," Circular A-94, revised October 29, 1992, p. 9.

<sup>12</sup> While Appendix C of the circular is revised annually, the updated discount rates only apply to "lease-purchase and cost-effectiveness analysis." They do not apply to "regulatory analysis or benefit-cost analysis of public investment."

<sup>13</sup> Including a historical series on total returns from stocks back to 1871 compiled by Robert Shiller.

<http://www.econ.yale.edu/~shiller/data.htm>

<sup>14</sup> Boardman et al, 2006, p 249.

compelling of which is that using a measure based on *average* returns to equities produces a discount rate that is too high because the return on the marginal investment is lower than the average return. In the bond market, the interest rate represents the marginal borrower's willingness to pay, and this should proxy the return on the marginal investment. Other reasons include (1) avoiding the problem of having to estimate the effective marginal corporate tax rate (because a firm can deduct the interest it pays to its bondholders before calculating its taxable income, it will equate its expected before-tax return on an investment with the before-tax rate it must pay on its bonds) and (2) returns to equity investments contain a premium for bearing the extra risk of holding equities which should not be included in the discount rate.<sup>15</sup>

Real yields on Moody's-rated corporate bonds can provide an appropriate proxy for the marginal rate of return on private capital. The annual return on AAA-rated corporate bonds has averaged a little over 3% for the past 90 years. Returns on BAA-rated bonds have been about a percent higher. The last 2 decades (a period of relatively predictable inflation) have seen slightly higher yields as shown in the table below. Overall, an estimate of **3.5%** appears reasonable for the Social Opportunity Cost of Capital.

**Table 2**  
**Average Annual Yields on Moody's AAA-Rated Corporate Bonds**

	Nominal	Real
1919-2010	5.9%	3.1%
1954-2010	7.2%	3.3%
1991-2010	6.6%	3.9%

Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis

Full disclosure: there is a wide range in estimates for the Social Opportunity Cost of Capital in the literature. Consistent with the 3.5% figure above, McGrattan and Prescott<sup>16</sup> estimate that the real after tax rate of return on U.S. reproducible capital has averaged approximately 4% over the period 1880-2002. On the high side is Harberger<sup>17</sup> who argues that the pre-tax rate of return on investment is at least 10% while Richard Zerbe and David Burgess<sup>18</sup> suggest a real rate of between 6% and 8% is appropriate. For the reasons outlined above, I believe the numbers at the lower end of the range to be more defensible.

The question still remains of how to use these two concepts in determining the appropriate discount rate. In much of the literature, the issue comes down to the source of funds for public investments. To the extent that transferring funds from the private sector displaces consumption, the social marginal rate of time preference is the appropriate concept for the discount rate. If private investment is displaced, the marginal pre-tax rate of return on private investment must be

<sup>15</sup> Again, the issue of how to account for risk and uncertainty is handled later in this paper.

<sup>16</sup> McGrattan, E.R. and E.C. Prescott. 2003. "Average Debt and Equity Returns: Puzzling?" Federal Reserve Bank of Minneapolis, Research Department, Staff Report Number 313.

<sup>17</sup> Harberger, A.C. 2010. On growth, investment, capital and the rate of return, in D.F. Burgess and G.P. Jenkins (eds.), Discount Rates for the Evaluation of Public Private Partnerships. Montreal and Kingston: McGill-Queen's University Press.

<sup>18</sup> Burgess, David F. and Zerbe, Richard O., 2011 "Appropriate Discounting for Benefit-Cost Analysis," *Journal of Benefit-Cost Analysis*.

brought into the calculation. Two methods are often suggested: the shadow price of capital method and the weighted average method.

*The Shadow Price of Capital.* The general consensus among economists appears to be that using the Shadow Price of Capital (SPC) is the conceptually superior method of bridging the gap between the social rate of time preference and the opportunity cost of capital. The basic idea is that a project's cost and benefits can be split into consumption flows and investment flows. Costs that are financed through taxes or rate revenues are assumed to displace consumption and are considered consumption flows. Debt-financed expenditures are assumed to displace investment (at least when the economy is at or close to full employment) and are classified as investment flows. Similarly, benefits are assumed to be either consumed or reinvested and are classified accordingly. Investment flows are then multiplied by the SPC to convert them to consumption equivalents which can then be discounted by the SMRTP along with the other consumption flows.

So far so good but the problem is that different sources suggest different formulas for calculating the SPC, different values for the inputs, and different rules for applying it.<sup>19</sup> In the literature reviewed, estimates for the SPC ranged from less than one to very large with most estimates falling between 1.1 and 4. Some sources just apply the SPC to a project's capital costs (or the portion of it that is debt-financed) while others insist on a much more complex process involving the project financing and the specific stream of interest and principal payments.<sup>20</sup> Still others are vague on exactly how the SPC is to be applied. Even if the same formula with the same input values is used, the different rules for applying the SPC lead to very different results.

Following the example of applying the SPC in Boardman et al, and using their formulas, input values<sup>21</sup> and application method, a project with an initial capital cost of \$3 million, financed over 30 years at 4% interest, and producing benefits of \$150,000 annually over 30 years would have a Net Present Value of \$370,000. This is the same result as would be obtained by using a single discount rate on all costs and benefits of 2%. However using different formulas and application methods would produce different results. Even assuming all the issues around determining the Shadow Price of Capital were settled, the discount rate implied by the SPC method would still vary from project to project due to different ratios of consumption to investment flows.

*The Weighted Average Approach.* Perhaps a simpler means of deriving a single social discount rate is to calculate a weighted average of the SMRTP and SOCC with the weights reflecting the proportion of project funding that displaces consumption and private investment, respectively. The standard assumption is that tax or revenue financed public expenditures displace consumption while debt-financed expenditures displace private investment. The latter assumes that issuing public debt competes for available investment funds causing the market interest rate to rise and "crowd out" marginal private investment. That however depends on the state of the economy. If, as is now the case, the economy is in the doldrums, operating far below capacity

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<sup>19</sup> See Boardman et al, 2006, pp 253-257; Zerbe, Richard O. and Dively, Dwight D., 1994, *Benefit Cost Analysis in Theory and Practice*, pp 208-287; Lind, Robert C., 1982, *Discounting for Time and Risk in Energy Policy*, pp 39-55; US Environmental Protection Agency - National Center for Environmental Economics Office of Policy, 2010, *Guidelines for Preparing Economic Analysis*, pp 6-10 – 6-13; Gomez-Ibanez, Jose, 1999, *Essays in Transportation Economics and Policy*, pp 170-173.

<sup>20</sup> In my judgment, the more complex method is the correct one.

<sup>21</sup> SOCC = 4.5%, SMRTP = 1.5%, Depreciation Rate = 10%, Reinvestment Rate = 17%, SPC = 1.33.

with high unemployment and low, zero or even negative real interest rates, there's little chance of public investment crowding out private investment. Under those conditions, debt-financed expenditures can be treated the same as tax and revenue financed expenditures, and assumed to displace consumption rather than investment.

The rationale for setting SPU-specific weights is as follows. Project costs are divided between capital and operations & maintenance (O&M). O&M is all revenue-financed and capital expenditures are *mostly* debt-financed. The water fund has a target of revenue financing at least 20% of capital expenditures while the target for the drainage and wastewater fund is 25% revenue financing.

Every project has a different mix of capital and O&M. Alternatives for the capital-intensive Morse Lake Pump Plant project had capital shares ranging from 60% to 90%. In order to provide a single discount rate for all projects, capital as a percent of total costs is assumed to average 75%.

Finally, the US economy has officially been in recession 16% of the time since 1945. This probably understates the actual duration of slackness in the economy. For example, the National Bureau of Economic Research only classifies the period December 2007 through June 2009 of the current "Great Contraction" as being in recession even though we are still in a time of low demand, high unemployment, and negative real interest rates. Perhaps a better indicator of when crowding out is unlikely to be a problem is the unemployment rate exceeding the "natural" or "full employment" rate by a margin of, say, 1%. That has occurred about one third of the time over the past 42 years. Rather than changing the discount rate as economic conditions change, we can average over the business cycle by assuming that crowding out of private investment occurs 67% of the time.

Combining the above produces the weights for the weighted average calculation as shown below. This produces an estimate of the social discount rate of **2.3%**.

Market Interest Rates	
(a) <b>Social Opportunity Cost of Capital (SOCC)</b>	<b>3.5%</b>
(b) <b>Social Marginal Rate of Time Preference (SMRTP)</b>	<b>1.5%</b>
Weight for Displaced Investment	
(c) Capital as a Percent of Total Project Cost:	75%
(d) Percent of Capital that is Debt-Financed:	80%
(e) Percent of Time with Crowding Out:	67%
(f) <b>Weight for Displaced Investment (c*d*e):</b>	<b>40%</b>
Weight for Displaced Consumption	
(g) <b>Weight for Displaced Consumption (1-f):</b>	<b>60%</b>
<b>Social Discount Rate (SDR) = (f*a + g*b) =</b>	<b>2.3%</b>

*Government Borrowing Rate.* Still another market-based discount rate approach is to simply use a public agency's cost of borrowing as its discount rate. This method is used by Tacoma Power and seems plausible because the rate reflects the actual cost of borrowed funds to the agency financing the project. The primary criticism by economists is that the Social Discount Rate should reflect the cost to society of foregoing current consumption or investment, not to the particular government agency. However, it's been suggested by at least one economist that in a world with high international capital mobility and thus little crowding out, net benefits for most public projects should be computed using the government borrowing rate as the discount rate.<sup>22</sup> For the record, the interest rate on SPU bonds over the last decade has averaged **2.1%** after adjusting for inflation.

## **Non-Market Based Approaches and Other Considerations**

*Optimal Growth Rate Approach (OGR).* This method was first suggested by Frank Ramsey<sup>23</sup> and it both rejects and predates the idea that social choices should reflect individual preferences inferred from market interest rates.<sup>24</sup> The OGR approach assumes that government policy makers seek to choose the amount of public investment that maximizes society's well-being now and in the future using a social welfare function that reflects two characteristics – impatience and egalitarianism. Society, like an individual, has a degree of impatience – a preference for present consumption over future consumption. Society also has an interest in making consumption flows on a per person basis more equal over time. Both of these contribute to a positive social discount rate ( $o$ ) which is calculated by adding the pure social rate of time preference ( $d$ ) to the product of the growth rate in per capita consumption ( $g$ ) and the absolute value of the rate at which the marginal value of that consumption decreases as per capita consumption increases ( $e$ ), i.e., a declining marginal utility of consumption:

$$o = d + ge$$

The problem of course is coming up with numbers for those parameters, especially  $d$  and  $e$ . Boardman et al suggest using 2.3% for  $g$  based on the average historical growth rate in real U.S. per capita consumption over the period 1947-2002. Extending the series to 2010 with recently available data also has the advantage of placing both endpoints of the series in the trough of a recession. This produces a slightly lower average growth rate of 2.1%. Prescott<sup>25</sup> argues for using a real growth rate of 2% as this was the average growth rate of the U.S. economy over the 20th Century. The same 2.0% average annual rate of per capita growth applies to the 100 year period 1910 through 2010.

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<sup>22</sup> Lind, Robert C. 1990. "Reassessing the Government's Discount Rate Policy in Light of New Theory and Data in a World Economy with a High Degree of Capital Mobility," *Journal of Environmental Economics and Management*.

<sup>23</sup> Ramsey, Frank, 1928. "A Mathematical Theory of Saving," *Economic Journal*.

<sup>24</sup> For the usual reasons, some of which have already been mentioned on page 3: imperfect capital markets, "irrational" consumer behavior, taxation and other distortions, non-existence of intertemporal markets for *public* (as opposed to private) goods.

<sup>25</sup> Prescott, E. 2002. "Prosperity and Depressions," *American Economic Review*, 92(2), 1-15.



Boardman et al recommend a value of 1 for  $e$ , splitting the difference between Brent<sup>26</sup> who suggested that  $e$  should be between 0 and 1, and Arrow et al<sup>27</sup> who argued that individual elasticities of marginal utility of consumption lie between 1 and 2.

Much more controversy concerns the value of  $d$ . Some including Ramsey himself, argue it's ethically indefensible to use a value greater than zero, as this discounts future generations' well-being relative to the present. However, Arrow<sup>28</sup> and others have countered that the practical implication of weighting all generations' welfare equally is to require extremely high rates of savings (more than half of income) from the current generation. Arrow suggests a figure of about 1% for  $d$ , which Boardman et al use in their calculations (and who wants to argue with Kenneth Arrow?). Plugging these values ( $d = 1\%$ ,  $e = 1$ , and  $g = 2\%$ ) into the equation for  $o$  produces an estimate for the social discount rate of **3%**.<sup>29</sup>

The underlying assumption here is that per capita consumption will continue to grow faster than the population and therefore, people will be richer in the future. This was a good assumption for the last century but may not be for the next. While it's possible that human ingenuity will find ways to overcome the impact of continuing population growth combined with climate change and the depletion/degradation of the resource base, it's also not inconceivable that the world's inhabitants will find themselves worse off in the next century. If that turns out to be the case, the appropriate discount rate using the OGR approach could be close to zero or even negative.

*Intergenerational Considerations and Constant vs. Time-Declining Discount Rates.* The implications of discounting at a constant rate over long periods of time – essentially reducing the present value of costs and benefits in the far future to insignificance – has prompted many economists and non-economists to question whether such discounting can be justified on either ethical or technical grounds. This issue often becomes most relevant when analyzing projects with long-lasting environmental impacts.

Some economists maintain that this is not an issue – costs and benefits for all projects should be discounted using a constant discount rate, even when they occur far in the future. This view is often associated with those who see the social discount rate as primarily representing the opportunity cost of capital. Lowering the social discount rate or having it decline over time results in economic inefficiency as funds are shifted from private projects to public projects that yield lower rates of return. Another common concern stems from the assumption that future generations will be better off than the present. Using a time-declining discount rate would then be a case of Robin Hood in reverse: taking from the (present) poor and giving to the (future) rich. Finally, a constant discount rate is undeniably computationally convenient and it avoids the problem of time inconsistency.<sup>30</sup>

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<sup>26</sup> Brent, R.J., 1994. Applied Cost-Benefit Analysis, Edward Elgar.

<sup>27</sup> Arrow, K.J., Cline, W.R., Maler, K-G., Munasinghe, M., Squitieri, R. & Stiglitz, J.E., 1995. "Intertemporal Equity, Discounting and Economic Efficiency," in Bruce et al. (Eds.), Climate Change, 1995, Cambridge University Press, 128-144.

<sup>28</sup> Arrow, K.J., 1995. "Intergenerational Equity and the Rate of Discount in Long-Term Social Investment," Paper given to the IEA World Congress.

<sup>29</sup> In a new article published in 2013, Moore, Boardman and Vining suggest slightly different values for the OGR model parameters:  $d=1\%$ ,  $e=1.35$ ,  $g=1.9\%$ , which results in a slightly higher estimate for the SDR of 3.5%

<sup>30</sup> Using a discount rate that varies with time means that if you redo a benefit-cost analysis in which nothing has changed except the passage of time, you may get a different result implying a different optimal decision.

On the other hand, there are a number of compelling arguments for using a time-declining discount rate. First is the ethical dilemma posed by constant rate discounting over long periods of time. Consider a scenario in which it is known that allowing CO<sub>2</sub> to continue accumulating in the atmosphere at current rates for another 20 years will set in motion an irreversible process having little immediate impact but causing a devastating shift in the world’s climate 400 years hence. It’s also known with certainty that this will cost society \$30 trillion (half of current world GDP).

**Table 3**  
**Present Values of \$30 Trillion 400 Years in the Future**

Discount Rate	Present Value
1%	\$560,000,000,000
2%	\$11,000,000,000
3%	\$220,000,000
4%	\$4,600,000
5%	\$100,000
6%	\$2,300
7%	\$53

That \$30 trillion cost has a present value of only \$11 billion discounted at 2%, \$220 million discounted at 3%, \$100,000 discounted at 5%, and just \$53 at 7%. Using a constant discount rate much in excess of 1 percent implies that it is not economically efficient for society to spend even a small amount today in order to avert a very costly environmental disaster, provided that the disaster occurs sufficiently far in the future. A time-declining discount rate avoids having to choose between ignoring very long term environmental consequences and not discounting at all.

Another ethical consideration is the fact that the various market interest rates used to estimate the social rate of discount reflect the preferences of people who are presently alive, not those not yet born. It can be argued that those of us in the current generation fail to appropriately account for the long-term impacts of our actions, since we won’t be around to enjoy or suffer them. Time-declining discount rates would be a move in the right direction to the extent that we undervalue the welfare of future generations.

From the relatively new field of behavioral economics comes much empirical evidence suggesting that individuals are themselves time inconsistent – making decisions that imply the use of high discount rates in the short term and low discount rates over long time horizons.<sup>31</sup>

For economists who feel that ethics and values are beyond the purview of the profession, there is also a technical reason for declining discount rates: uncertainty – not uncertainty about the magnitudes of a project’s costs and benefits, but uncertainty around the parameters determining

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<sup>31</sup> Frederick, Shane, Loewenstein, George, and O’Donoghue, Ted, 2002, “Time Discounting and Time Preference: A Critical Review” *Journal of Economic Literature*.  
Groom, Ben, Hepburn, Cameron, Koundouri, Phoebe, and Pearce, David, 2005, “Declining Discount Rates: The Long and the Short of it,” *Environmental & Resource Economics*.

the discount rate itself. Because of the nature of discounting, discount rates at the very low end of the uncertainty range will dominate more and more as the time horizon gets longer.

To see why, consider just the uncertainty about the future rate of per capita economic growth,  $g$ , in the OGR model. For ease of exposition, let's assume uncertainty involving just two possibilities, a 50% chance of rapid growth, say 4% a year (due to hoped-for technological breakthroughs) and a 50% chance of 0% growth (productivity growth just able to keep up with population). These two growth rates imply alternative discount rates of 1% and 5%, each with a probability of 50%.

The expected present value of \$1 billion dollars 400 years from now is not \$7,000 which is what would be obtained by discounting at 3%, the average of the two rates. Rather, one must take the weighted average of the two present values obtained by using the two different rates:

$$\begin{aligned}
 PV &= 0.5 \times \frac{\$1,000,000,000}{(1+0.01)^{400}} + 0.5 \times \frac{\$1,000,000,000}{(1+0.05)^{400}} \\
 &= \$9,341,583 + \$1.67 = \$9,341,585
 \end{aligned}$$

The expected present value is \$9,341,585. This is equivalent to using a single, certain discount rate of approximately 1.18%. In effect, the larger discount rate almost completely discounts itself out of the average (down to \$1.67 in this case). This effect grows over longer and longer time horizons, resulting in a time-declining schedule of discount rates. In the distant future, only the very lowest possible rate matters; all the higher rates discount themselves into insignificance.

Boardman et al define *intragenerational* projects as those whose main effects occur within 50 years and *intergenerational* projects as those with significant effects beyond 50 years. They suggest, citing Newell and Pizer,<sup>32</sup> the use of time-declining discount rates such as those shown in the table below:

**Table 4 – Time Declining Discount Rates**

Years	Discount Rates Using:	
	OGR Approach	Market-Based*
< 50	3.0%	2.3%
50-100	2.2%	1.5%
100-200	1.5%	1.0%
200-300	0.5%	0.5%
> 300	0.0%	0.0%

\* Weighted Average of SMRTP & SOCC

Thus, a project with a \$100,000 benefit in year 250 would be discounted as follows (using the OGR approach):

<sup>32</sup> Newell, R.G. and Pizer, W.A., 2003, "Discounting and Distant Future: How Much Do Uncertain Rates Increase Valuations?" *Journal of Environmental Economics and Management*.

$$PV = \frac{\$100,000}{(1+0.030)^{50} \times (1+0.025)^{(100-50)} \times (1+0.015)^{(200-100)} \times (1+0.005)^{(250-200)}} = \$1,167^*$$

Using this time-declining schedule of discount rates over 250 years is equivalent to discounting at a constant **1.8%**.

*And Now for Something Completely Different – Gamma Discounting.* As has already been pointed out, there are many unresolved issues, both philosophical and practical, in determining the appropriate rate of discount for benefit-cost analysis. I am not the first nor will I be the last economist to express frustration over this but Martin Weitzman<sup>34</sup> takes a different approach. Like many others, he observes that no consensus exists, has ever existed, or is likely to exist in the future about what actual rate of interest to use for social discounting. The list of controversies on which professional opinion is divided is long and much of it does just come down to a matter of judgment, on which fully informed and fully rational individuals might be expected to differ. So Weitzman just conducts a survey. He asked over 2000 economists to give their ‘professionally considered gut feeling’ on what is the appropriate discount rate for evaluating long term public investments with environmental benefits such as climate change mitigation. The responses approximately followed a gamma distribution ranging from -3% to +27% with a mean of 4%, mode of 2% and standard deviation of 3 percentage points. Interpreting this distribution as reflecting the state of uncertainty around the parameters determining the discount rate, Weitzman concluded (using the same logic as explained in the previous section) that the discount rate should decline over time as shown, below.

**Table 5 – Time Declining Discount Rates Implied by Gamma Distribution**

Time from Present	Description	Marginal Discount Rate
1-5 years	Immediate Future	4%
6-25 years	Near Future	3%
26-75 years	Medium Future	2%
76-300 years	Distant Future	1%
>300 years	Far-Distant Future	0%

Depending on the temporal distribution of costs and benefits and over how long a time horizon, this declining scale of discount rates is equivalent to a constant discount rate in the range of **1.2% to 3.4%**. The concept of constant rate equivalents to time declining discounts rates is discussed in more detail later in this paper.

*Risk and Uncertainty*

There still remains the matter of risk – that is, uncertainty around the estimates of a public project’s net benefits, (but not uncertainty around the correct value of the discount rate) – and whether a risk premium should be included in the discount rate. Risk of nonpayment was one of the reasons given at the beginning of this paper as to why one should expect to be charged interest on a loan of \$10. The same logic applies to the higher interest rates paid by junk bonds. So while including a risk premium in the social discount rate does have an intuitive appeal, most

\* Note that the present value of \$100,000 in 250 years discounted at a constant 3% is only \$62.

<sup>34</sup> Weitzman, Martin, 2001, “Gamma Discounting,” *American Economic Review*.

of the economic literature reviewed argues the opposite<sup>35</sup> (though of course, there is not complete agreement on this point in the profession<sup>36</sup>).

Two common arguments for why market based estimates of the social discount rate should use the return on a riskless asset are (1) the government can effectively reduce the non-systematic risk that individual citizens bear to zero by pooling risks across the entire population and (2) it is appropriate to separate the issues of risk and discounting by converting net benefit flows to certainty equivalents prior to discounting at a risk-free rate. I find the last point particularly persuasive for practical reasons. If a risk premium is to be added to the discount rate, there is the problem of determining how big it should be. A risk premium is also a pretty blunt instrument. Much information about the risk profile of a project would be lost by reducing it down to a single number.

But even more perplexing, consider a benefit cost analysis involving three alternatives. For the first one, there's a high degree of uncertainty around both the benefits and the costs. The cost of the next is known with certainty but the benefits are uncertain and also correlated with the ups and downs of the broader economy. Finally the costs and benefits of the last alternative are known with a high degree of certainty. The risks of the three alternatives are very different so how could they be reduced to single risk premium? They would have to have different risk premiums with the result that each alternative would have to be evaluated using a different discount rate.

It would be much better to account for the risk in the analysis outside of the discount rate. This could be done by placing uncertainty bounds around the mean estimates of the costs and benefits and evaluating the impact of risk in the sensitivity analysis, using Monte Carlo simulations if need be.

[*Postscript on Risk*: The above section on risk generated considerable discussion among SPU economists about whether a risk premium should be included in SPU's discount rate and if so, what it should be. A series of meetings was held on the topic and although some progress was made towards consensus, it also became clear that the issue is more complicated than originally thought. The common conception is that risk refers to uncertainty around the costs and/or benefits of an individual project and thus its net rate of return. The more uncertainty/variability around its potential returns, the riskier a project is considered. And assuming that people are risk averse, a riskier project would seem to require a higher risk premium on the discount rate used to evaluate it, i.e., a higher expected rate of return to compensate for the risk. This however turns out to be the wrong way to think about it. As pointed out by Lind,<sup>37</sup> it's not the riskiness of the individual project that's relevant, it's how it affects the riskiness of the whole portfolio. If SPU were to handle risk by applying a risk premium to the discount rate used to evaluate a project,

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<sup>35</sup> See Arrow, K.J. & Lind, R.C., 1970, "Uncertainty and the Evaluation of Public Investment Decisions," *American Economic Review*; Boardman et al, 2006; Grant, S. and Quiggin, J., 2003, "Public Investment and the Risk Premium for Equity," *Economica*; Howarth, Richard B., 2009, "Rethinking the Theory of Discounting and Revealed Time Preference," *Land Economics*; Peacock, Bruce, 1995, "The Appropriate Discount Rate For Social Policy Analysis: Discussion and Estimation," Issue Paper, U.S. Dept. of the Interior, Office of Policy Analysis.

<sup>36</sup> Burgess, David F. and Zerbe, Richard O., "Appropriate Discounting for Benefit-Cost Analysis," *Journal of Benefit-Cost Analysis*, 2011.

<sup>37</sup> Lind, Robert C., *Discounting for Time and Risk in Energy Policy*, Resources For the Future, 1982, 60-61.

whether that premium should be positive, negative or zero depends on whether the project increases, decreases or has no effect on total portfolio risk. This implies that:

- A positive risk premium should only be applied if the risk for an individual project is positively correlated with the total portfolio.
- If project risk is negatively correlated with the whole portfolio, (say construction costs go down in a recession, the benefits provide more value during hard times, or the project provides some form of insurance) then a negative risk premium would be applied.
- If the risks are uncorrelated or it's too difficult and complicated to determine the correlation, no risk premium should be applied to the discount rate.

Therefore, a high degree of uncertainty around the expected return to an individual project would not, in itself, justify a positive risk premium. In fact, individuals, private businesses and public agencies often apply *negative* risk premiums to investments with highly variable and uncertain outcome and, as a result, still choose to make those investments even when their expected returns are negative. They do this because these investments pay off well when the rest of their portfolios are doing poorly, and vice versa. Investors accept the low, even negative returns on these investments because they reduce their total portfolio risk. One example: homeowner's insurance.

This definition of project risk as the covariance between the outcome uncertainty around the individual project and that of the public portfolio was accepted by the economists. It was also agreed that the "public portfolio" from the standpoint of SPU should be defined as the regional economy as a whole. However to actually apply this concept to business cases, both the sign and magnitude of the covariance between the returns of an individual project alternative and the total portfolio would have to be determined as well as how to convert that information into an actual risk premium. Here there was also agreement among the economists, but it was that this would be an exceedingly difficult task that would consume more analytical resources than SPU has available. This writer therefore recommends that our default assumption be that project risk is uncorrelated with the larger economy and that the appropriate risk premium is zero. The issue of risk and uncertainty should continue to be dealt with outside of the discount rate.]

*Discount Rates Used by Other Departments and Government Agencies.* Finally, a quick non-random survey of discount rates used by other entities revealed the following:

Seattle City Light	3%
City of Seattle Dept of Finance	2% *
King County	7% **
Sound Transit	3%
Puget Sound Regional Council	2% - 4% ***
Tacoma Power	Cost of borrowing
Port of Seattle	No formal policy
State of Washington	No formal policy

\* 6% for "enterprise" projects. Memo from Greg Hill to Dwight Dively, date unknown

\*\* King County is currently reassessing its discount rate policy

\*\*\* Based on Treasury bond interest rates

Note: All of the above are expressed in constant, inflation-adjusted rates.

At the national level, most federal agencies doing benefit-cost analysis must follow the discount rate policy set by the U.S. Office of Management and Budget. In its Circular No. A-94, the U.S. OMB makes a distinction between “public investment and regulatory analysis” and “cost-effectiveness, lease purchase, and related analyses.” For the former it recommends a **7%** real (inflation adjusted) discount rate using an opportunity cost of capital argument. (This appears to be the basis for King County’s current policy and the discount rate suggested for “enterprise” projects by Seattle’s Department of Finance.) As far as can be determined, the OMB Circular has not been updated since 1992. However, discount rates for “cost-effectiveness, lease purchase, and related analyses” are revised annually, most recently in December 2012. These rates are based on Treasury bond rates and *increase* with time, ranging from **-1.4%** (!) for projects of 3 years or less to **1.1%** for projects of 30 years or more.

A notable exception is the US Army Corps of Engineers which each year sets its discount rate for project evaluations to the average annual *nominal*<sup>38</sup> yield on government securities with 15 years or more to maturity. The rate currently used by the Corps is **3.5%**.<sup>39</sup> This is equivalent to a real rate of **1.8%**.

### Convergence

This paper has briefly described many of the different conceptual approaches to discounting and some of the issues involved in choosing an appropriate numerical value for the discount rate. The discounting concepts and my best estimate of their numerical values are shown in the table below.

**Table 6 – Rates Implied by Different Discounting Concepts**

Discounting Concept	Rate
Social Rate of Time Preference	1.5%
Marginal Return on Private Capital*	3.5%
Synthesis of the above using:	
Shadow Price of Capital	Varies
Weighted Average Approach	2.3%
Government Borrowing Rate	2.1%
Optimal Growth Rate Approach	3.0%
Time Declining Rates**	0% -4%

\* or Opportunity Cost of Capital

\*\* including Gamma Discounting

What becomes immediately clear is that, despite the many unresolved issues, all of these values are significantly less than the baseline discount rate of 5% now used by SPU. In addition, other

<sup>38</sup> Oddly, the Corps is required by law to use *nominal* discount rates (*not* adjusted for inflation) but apply them to *real* costs and benefits (measured in inflation-adjusted dollars). This is just wrong and of course becomes more problematic the higher the inflation rate.

<sup>39</sup> U.S. Army Corps of Engineers, Memorandum for Planning Community of Practice, October 17, 2013.

public agencies in the region use discount rates between 2% and 4% (except for King County which is currently reassessing its discount rate policy). It should therefore be uncontroversial to recommend that SPU lower the discount rate it uses for benefit-cost analysis. But by how much should the value of the discount rate be lowered and should it be constant over time or declining? For reasons outlined below, my recommendation is that SPU adopt a constant discount rate of **2.2%** with low and high values of **1.5%** and **3.5%** for sensitivity analysis.

*Constant vs. Time Declining.* While declining discount rates are probably more conceptually correct, a time invariant discount rate is recommended for several practical reasons. Using declining discount rates is more critical if a project has really large costs or benefits that will occur in the distant future, (for example, decommissioning costs for nuclear power plants along with costs of long term radioactive waste storage). However, most SPU projects have costs and benefits that occur more evenly over time or occur earlier rather than later in the project life-cycle. Declining rates are also more important for projects with very long time horizons but most projects evaluated by SPU have a time horizon of 100 years or less. Finally, a constant rate is computationally much simpler than time-declining discount rates.

In the table below, different schemes of time declining discount rates (for the gamma distribution, optimal growth rate and weighted average approaches – see Tables 4 and 5) have been converted to constant discount rate equivalents. The conversions are different depending on what’s assumed about the net benefit flows (a lump sum at the end versus an even flow over the project life). Different time horizons and different flow patterns produce different constant rate equivalents.<sup>40</sup>

**Table 7 – Constant Rate Equivalents for Time Declining Discount Rates**

Time Horizon	Constant Rate Equivalents					
	Gamma Distribution		Optimal Growth Rate		Weighted Average	
	Lump Sum	Flow	Lump Sum	Flow	Lump Sum	Flow
50 years	2.6%	3.0%	3.0%	3.0%	2.3%	2.3%
100 years	2.0%	2.7%	2.6%	2.9%	1.9%	2.2%
200 years	1.5%	2.3%	2.0%	2.7%	1.4%	2.0%
300 years	1.3%	2.2%	1.5%	2.6%	1.1%	1.8%

*Just Give Me a Number.*<sup>41</sup> Given the many controversies associated with defining the discount rate, there’s a remarkable convergence in the actual values implied by the different approaches. The overall range is between 1.5% and 3.5% and I recommend focusing on the weighted average approach slightly adjusted to reflect the constant rate equivalent for an even flow of net benefits over a 100 year time horizon: **2.2%**. Once again, the weighted average approach is a compromise between the Social Rate of Time Preference and Opportunity Cost of Capital concepts taking into account SPU’s average mix of revenue and debt financing as well as the

<sup>40</sup> For example, a constant annual flow of net benefits discounted at 2.3% per year for the first 50 years, at 1.5% for the next 50 years, and at 1.0% for the next 100 years has the same present value as that same flow discounted at a constant 2.0% over 200 years. Using the same declining discount rate schedule on a single lump sum payment in 200 years has the same present value as that payment discounted at a constant 1.4%

<sup>41</sup> The title of an article by Boardman et al (plus Mark Moore) based on the Social Discount Rate Chapter of their textbook on Cost-Benefit Analysis.



percent of time public investment might be expected to crowd out private investment. In my opinion, this method is superior to the Optimal Growth Rate approach because it requires less arbitrary assumptions about the parameters that determine the numerical value of the discount rate. The 2.2% rate is also very close to SPU's cost of borrowing.

The low and high discount rates recommended for use in sensitivity analysis are **1.5%** and **3.5%**. The lower figure is equal to the estimate of the Social Marginal Rate of Time Preference and is appropriate if it is assumed that a public project would do very little or no crowding out of private investment (i.e., when the economy is in recession or periods of slow growth with high unemployment). The 1.5% rate is also close to the constant rate equivalents of time declining discount rates applied to projects with significant long term net benefits over time horizons of a hundred or more years. For that reason, it is suggested that sensitivity analysis for such projects put more emphasis on the implications of discounting at the low end of the scale rather than the high.

The higher discount rate is equal to the estimate of the Social Opportunity Cost of Capital and is appropriate if it is assumed that a public project would mostly or fully crowd out private investment, (when the economy is booming and unemployment is low). It is also in the neighborhood of the discount rates implied by the Optimal Growth Rate approach and the constant rate equivalents for short time horizon projects (25 years or less) using the time declining Gamma Distribution discount rates. It is therefore suggested that sensitivity analysis for projects with shorter time horizons being initiated during economic good times put more emphasis on the sensitivity analysis results of discounting at the high end of the range.

For reasons outlined earlier, the recommended discount rates do not include risk premiums. It is therefore suggested that SPU develop better methods for evaluating the uncertainty around a project's net returns. Specifically, it would be an improvement over current practices to consistently define uncertainty bounds around the mean estimates of a project's costs and benefits and evaluate the impact of these risks in the sensitivity analysis.

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